

Runaway electrons in disruptions: sliding and screening

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on behalf of the
Plasma theory group
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① Starting remarks

② Sliding

③ Screening



Runaway team



Ola Embréus

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Linnea Hesslow

PhD student



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George Wilkie

Postdoc

- Ola: Close collisions, Bremsstrahlung
- Linnea: Partial screening effects
- Mathias: Synthetic synchrotron diagnostics
- George: Self-consistent electric field

Tools available for runaway studies at Chalmers

- 0D2P relativistic Fokker-Planck solvers
 - **CODE** – runaway electrons, linearized collision operator
 - synchrotron radiation
 - Bremsstrahlung
 - effect of partial screening **NEW!**
 - Rosenbluth-Putvinskii, Chiu-Harvey, Boltzmann avalanche operator
 - **NORSE** – nonlinear collision operator **NEW!**
 - **CODION** – runaway ions
- Radiation
 - **SOFT** – synthetic synchrotron diagnostics **NEW!**
 - **SYRUP** – synchrotron spectra

Outline

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② Sliding

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NORSE: NO-n-linear Relativistic Solver for Electrons

Motivation

- The more runaways, the bigger the problem
- Existing tools break down when more than a few % runaways
- Such RE densities obtainable in experiments

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Features

- 2D in momentum space, no spatial dependence
- Full Braams & Karney collision operator
- Arbitrary electric field strengths
- Radiation reaction
- Time-dependent plasma parameters

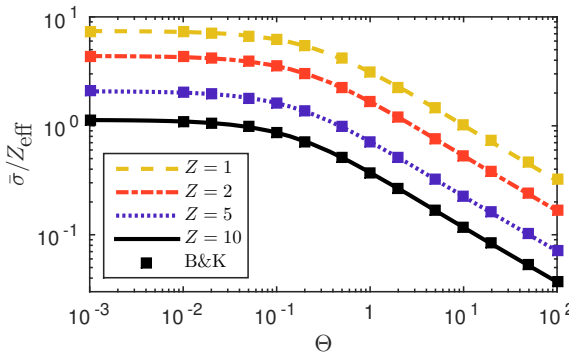


Benchmark: relativistic weak-field conductivity

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 - weak-field
 - large T range
 - same collision operator

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- Braams & Karney list conductivities
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- NORSE reproduces these perfectly



$\bar{\sigma}$: normalized conductivity

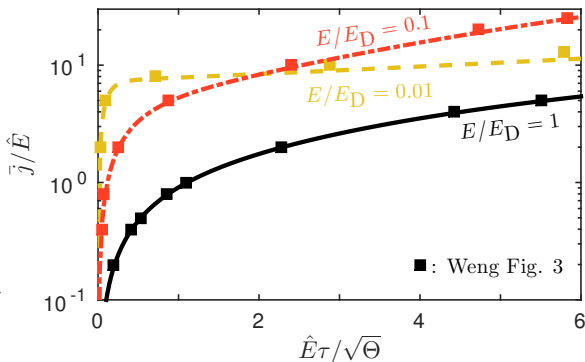
$$\Theta = T/m_e c^2$$

Benchmark: conductivity in strong fields

- Comparison to Weng et al. [PRL 100, 185001 (2008)]
- They calculate modified Spitzer conductivity in strong E field
- Non-relativistic
- Nice agreement!

(Numerical heating in Weng's data for

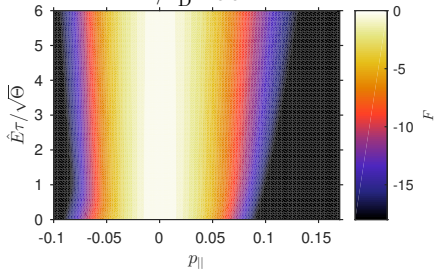
$E/E_D = 0.01$)



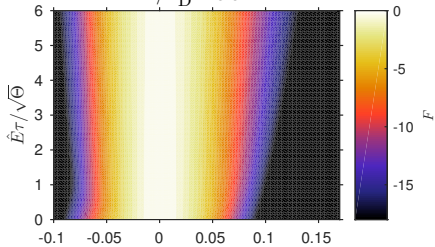
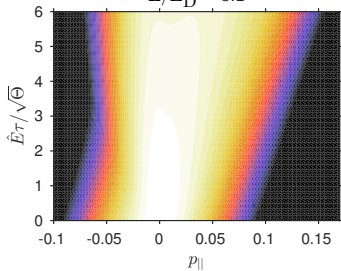
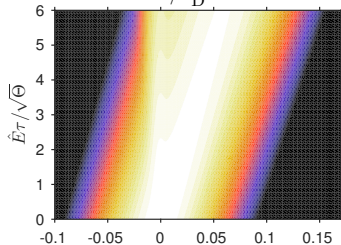
\bar{j}/\hat{E} : normalized conductivity

$\hat{E}\tau/\sqrt{\Theta}$: normalized time

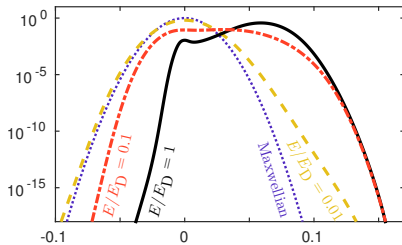
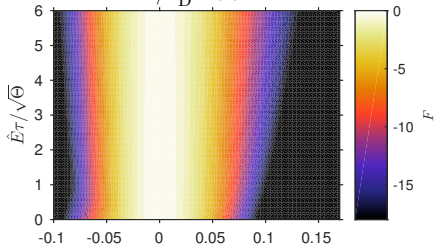
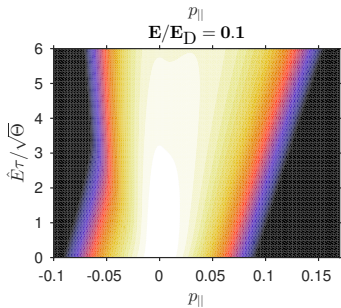
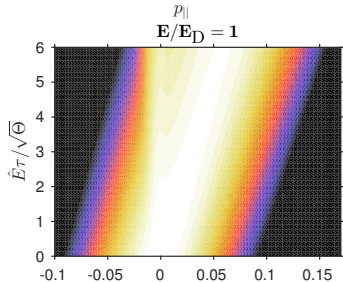
Distribution evolution

 $E/E_D = 0.01$ 

Distribution evolution

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 $E/E_D = 0.1$

 $E/E_D = 1$


Distribution evolution

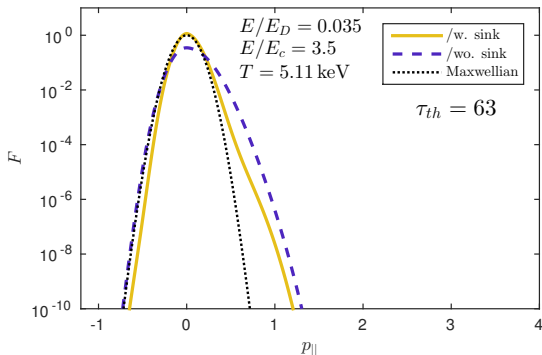
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Bulk heating

- E field is a source of heat!
 - Must be removed in a linear treatment
 - Automatically accounted for in NORSE
- In practice bulk keeps temperature or even cools – a heat sink is useful

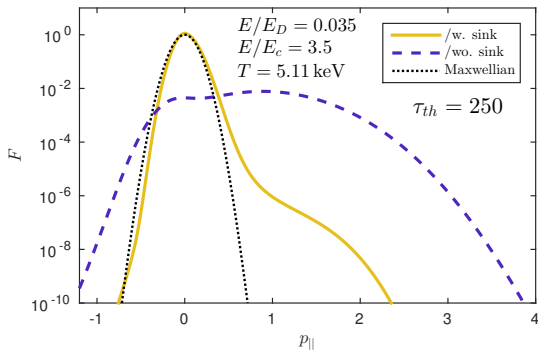
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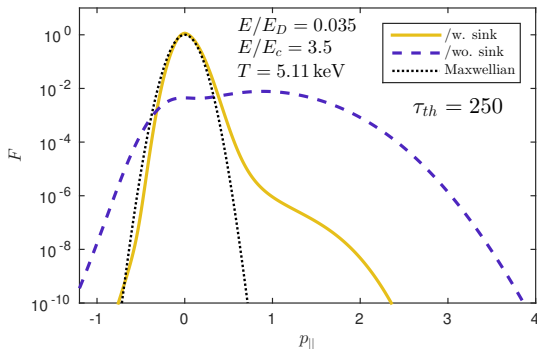
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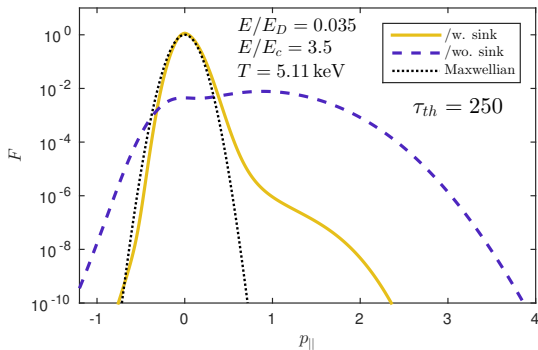
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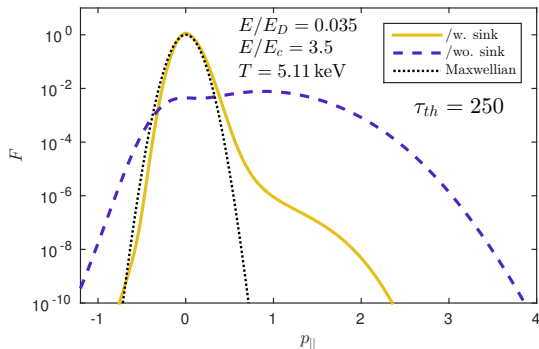
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Current evolution and transition to slide-away is highly sensitive to the details of the sink!

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Slide-away: Net parallel force experienced by electrons is positive in the entire momentum space.

Electric field

An ITER-like scenario calculated by GO [Smith et al (2006)]

- GO: generation of runaway electrons coupled to a diffusion equation for the electric field.

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial E}{\partial r} \right) = \mu_0 \frac{\partial}{\partial t} (\sigma_{\parallel} E + n_r e c)$$

and

$$\frac{\partial n_r}{\partial t} = \left(\frac{\partial n_r}{\partial t} \right)^{Dreicer} + \left(\frac{\partial n_r}{\partial t} \right)^{avalanche}$$

- $T_e^{final} = 10$ eV, $B = 5.3$ T, $Z_{eff} = 1$,
 $j_0 = 0.62$ MA/m², thermal quench time 1 ms.

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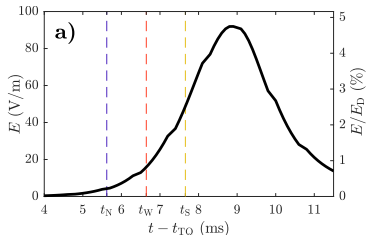
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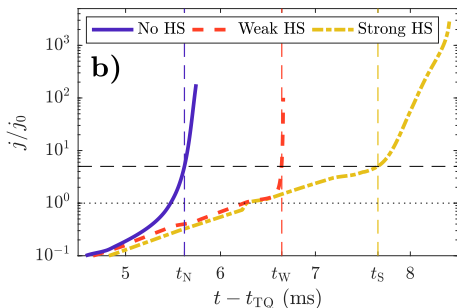
Electric field in V/m and normalized to the Dreicer field after the thermal quench.

Transition to slide-away depends on the heat-sink

- **No heat sink:** all energy supplied by the electric field remains.
- **Weak heat sink:** the energy-removal rate of the heat sink is restricted to 0.5 MW/m^3
- **Strong heat sink:** keep the bulk temperature at 10 eV; any excess heat in the bulk region is removed

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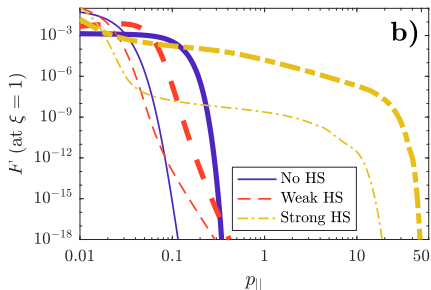
Normalized current density in the different heat-sink scenarios. Current density becomes half of the original at t_N (no HS), t_W (weak HS) and t_S (strong HS).

Runaway electron population

- Maximum particle energies depend on the heat-sink scenario.
 - **No HS** and **weak HS**: particle do not reach relativistic energies
 - **Strong HS**: particle energies of 22 MeV are reached just before slide-away.

Runaway electron population

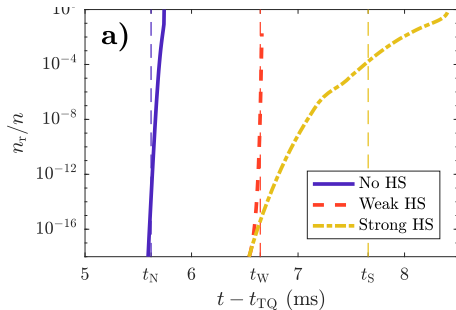
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Tail of the parallel electron distribution. Thin lines f at t_N (no HS), t_W (weak HS) and t_S (strong HS), and thick lines f immediately before the transition to slide-away.

Runaway electron population

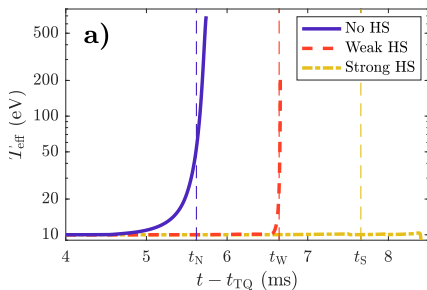
- Maximum particle energies depend on the heat-sink scenario.
 - **No HS** and **weak HS**: particle do not reach relativistic energies
 - **Strong HS**: particle energies of 22 MeV are reached just before slide-away.
- In the **strong HS** case the n_r/n grows more slowly and the runaways have time to reach high energies.



Runaway fraction

Feedback loop

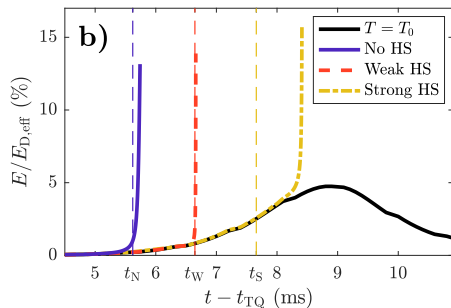
- Collisional friction is lower in a hotter distribution
- Dreicer field is $\propto 1/T$.
- For a given field strength E/E_D increases as the bulk heats up.
- Decreasing n_{bulk} also leads to a positive feedback.
- Eventually the friction becomes low enough that the parallel balance of forces becomes positive everywhere: Slide-away!



Effective temperature of the bulk population

Feedback loop

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Effective normalized E-field strength

Summary

NORSE [Stahl et al CPC (2017)]

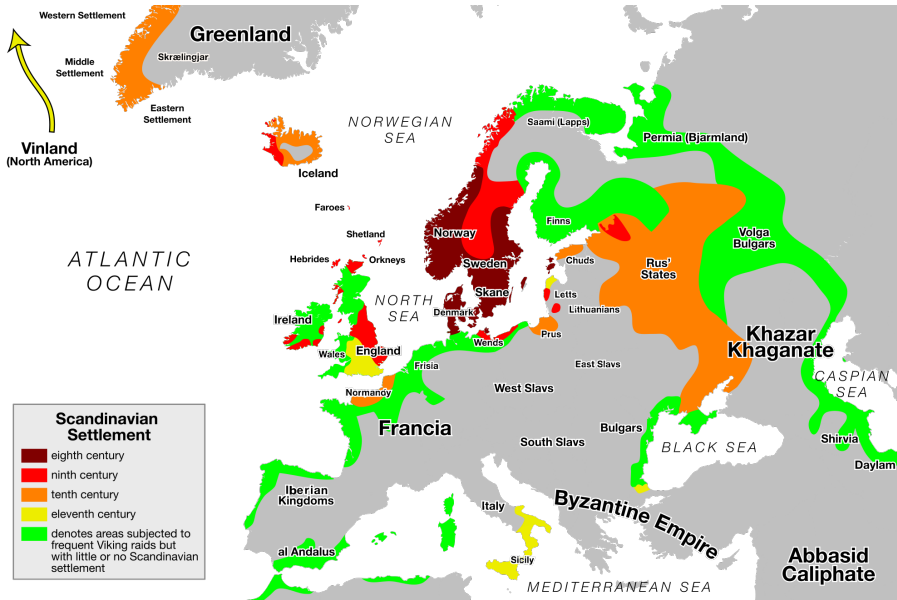
- Relativistic, non-linear electron dynamics
- Radiative effects, time-dependent scenarios
- Efficient, freely available

Non-linear effects

- Conductivity different from Spitzer for strong fields
- Large heating of electron bulk by parallel E -field
- Slide-away at much weaker electric fields than previously expected.

Heat-sink

- Severity of disruptions can be affected by the properties of heat sink.



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Effect of partial screening

- Disruption mitigation via material injection: typically $n_Z > n_D$.
- In the cold post-disruption plasma, impurities are weakly ionized.
- Collision frequencies for fast electrons are expected to be enhanced.

Previous work

- **Elastic collisions:** Thomas–Fermi theory (limited to intermediate distances from the nucleus, and does not capture the shell structure of the ion): [Kirillov et al Fizika Plazmy (1975)] and [Zhogolev and Konovalov VANT (2014) **in Russian**]
- Kinetic simulations in [Aleynikov et al, IAEA proceedings 2014] refers to [Zhogolev& Konovalov] for details.
- **Inelastic collisions:** Rosenbluth–Putvinski rule of thumb: half of the bound electrons [Rosenbluth and Putvinski, NF (1997)]
- **Stopping-power formula for inelastic collisions** was used in a test-particle approach in [Martin-Solis et al PoP (2015)].

Modelling of the effect of partial screening

- Generalized collision operator including the effect of partial screening

$$C_{test}^e = \nu_D \mathcal{L}(f_e) + \frac{1}{p^2} \frac{\partial}{\partial p} \left[p^3 \left(\nu_S f_e + \frac{1}{2} \nu_{\parallel} p \frac{\partial f_e}{\partial p} \right) \right]$$

- Model elastic collisions quantum-mechanically using density functional theory.
- Using kinetic simulations demonstrate the effect of partial screening on the distribution function, current decay and critical electric field.
- Analytical expression of the enhanced critical electric field.

[Hesslow et al, PRL (2017)]

Effect of partial screening

- Definitions
 - **Complete screening:** the electron interacts only with the net ion charge
 - **No screening:** the electron experiences the full nuclear charge
- **Elastic collisions**
 - Interaction strength proportional to the charge squared.
 - No screening enhances the interaction strength by a factor $X^2 = (Z/Z_0)^2$, where Z_0 is the ionization state and Z is the charge number of the nucleus.
- **Inelastic collisions** (leading to excitation of the ion)
 - Increase the effective electron density of the plasma, as experienced by the fast electron.
 - The rate of e-e collisions will be an order X larger.

Elastic collisions ν_D^{ei}

Cross section in Born approximation, valid for $v/c \gg Z\alpha$

$$\frac{d\sigma_{ej}}{d\Omega} = \left(\frac{r_0^2}{4p^4} \right) \left(\frac{\cos^2(\theta/2)p^2 + 1}{\sin^4(\theta/2)} \right) |Z_j - F_j(\mathbf{q})|^2$$

Form factor: $F_j(\mathbf{q}) = \int \rho_{e,j}(r) e^{-i\mathbf{q}\cdot\mathbf{r}/a_0} d\mathbf{r}$

$q = \frac{2p}{\alpha} \sin(\theta/2)$, $p = \gamma \frac{v}{c}$, Z : atomic number, Z_0 : net charge

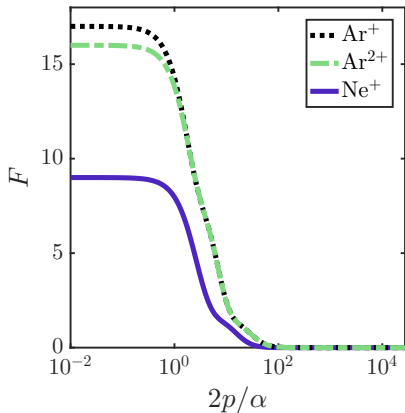
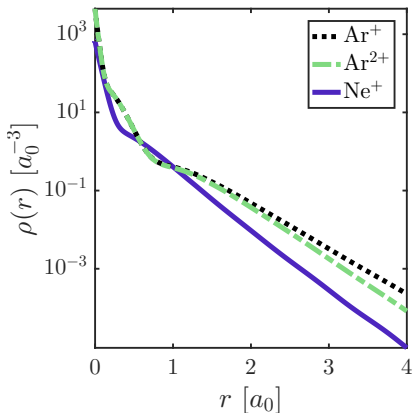
Limits:

Low energy $|Z - F| \rightarrow Z_0$: **complete screening** (usual case)

High energy $|Z - F| \rightarrow Z$: **no screening** (interaction with nucleus)

Elastic collisions: density and form factor


From density functional theory (DFT)



Elastic collisions ν_D^{ei}

$$\nu_D^{ei} = \nu_{D,CS}^{ei} \left(1 + \frac{1}{\sum_j n_j Z_{0,j}^2} \sum_j n_j Z_{0,j}^2 \frac{g_j(p)}{\ln \Lambda} \right)$$

completely screened
collision frequency




DFT simulations

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completely screened
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Full formula

$$g_j(p) = \int_0^1 \left(\frac{[Z_j - F_j(q)]^2}{Z_{0,j}^2} - 1 \right) \frac{dx}{x}$$

DFT simulations

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TF-DFT model

$$g_j(p) = \frac{2}{3} (X_j^2 - 1) \ln(y_j^{3/2} + 1) - \frac{2}{3} \frac{(X_j - 1)^2 y_j^{3/2}}{y_j^{3/2} + 1}$$

$$X_j = Z_j / Z_{0,j}$$

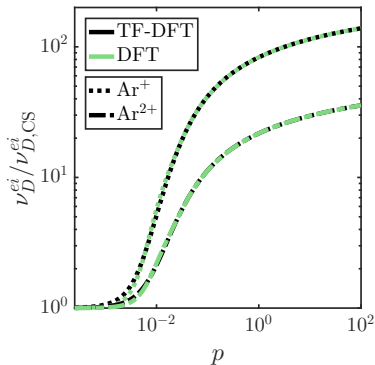
effective length a_j

$$y_j = 2a_j p / \alpha$$

DFT simulations

Enhancement of deflection frequency ν_D^{ei}

- Compare to completely screened
- Excellent agreement between analytical model (TF-DFT) and full DFT
- Significant effect already at $p \sim p_c \sim 0.1$
- $p \gg 1$: $\nu_D^{ei}/\nu_{D,CS}^{ei} \sim (Z/Z_0)^2 \sim 10^2$
- Parameters: $T = 10$ eV,
 $n_{Ar^+} = 10^{20} \text{ m}^{-3}$



$$p = \gamma \frac{v}{c}, \quad E = 10 \text{ MeV} \leftrightarrow p = 20.$$

Inelastic collisions ν_S^{ee}

Bethe stopping power formula (matched with low energy asymptote)

$$\nu_S^{ee} = \nu_{S,cs}^{ee} \left\{ 1 + \sum_j \frac{n_j N_{e,j}}{n_e \ln \Lambda} \left[\frac{1}{k} \ln(1 + h_j^k) - \beta^2 \right] \right\},$$

$$h_j = p\sqrt{\gamma - 1}/I_j, \quad k = 5, \quad \beta = v/c$$

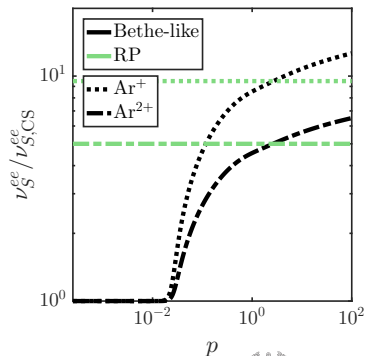
I_j mean excitation energy [Sauer et al, Advances in Quantum Chemistry 2015]

- Rosenbluth–Putvinski rule of thumb:

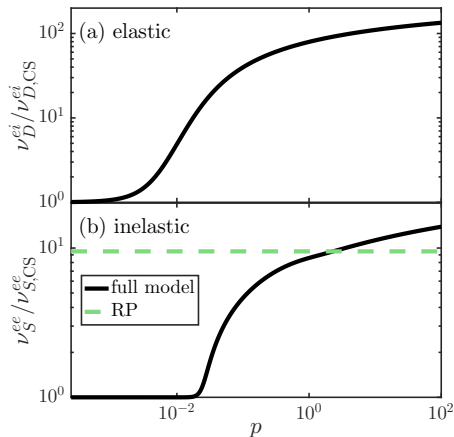
$$\nu_{S,rp}^{ee} \approx \nu_{S,cs}^{ee} \left(1 + \frac{1}{2} \sum_j \frac{n_j}{n_e} N_{e,j} \right),$$

where N_e is the number of bound electrons.

- RP rule of thumb leads to greater enhancement than the full formula up to $p \simeq 1$.



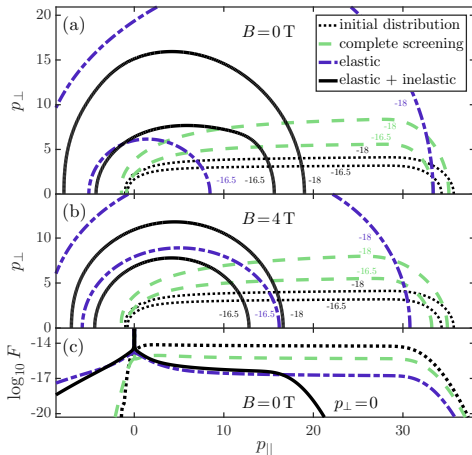
- Enhancement due to elastic collisions kicks in for lower momenta and is larger for high momenta than the corresponding one for inelastic collisions.



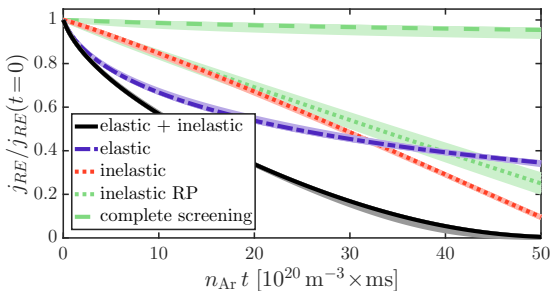
Parameters: $T = 10$ eV, $n_{\text{Ar}^+} = 10^{20} \text{ m}^{-3}$

Effect on distribution function

- Implemented in CODE.
- Collisional deceleration of initial beam-like distribution.
- Contours of $\log_{10}(F)$,
 $F = (2\pi m_e T)^{3/2} f_e / n_e$
- Parameters: 25 ms collisional deceleration $T = 10$ eV,
 Ar^+ , $n_{\text{Ar}} = n_{\text{D}} = 10^{20} \text{ m}^{-3}$



Current decay

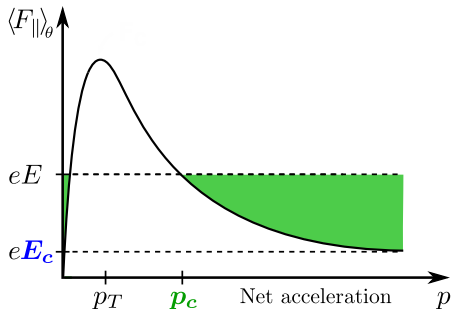


- Same initial distribution as previous figure.
- Decay time is proportional to $1/n_{Ar}$ for $n_{Ar} \gtrsim n_D$.
- Bands represent $n_{Ar} \in [0.5 n_D, 100 n_D]$.
- RP model underestimates the decay rate and shows a different current evolution.

Critical electric field

- Important for generation and decay
- Constant $\ln \Lambda$ and no screening or radiation effects:

$$E_c = \frac{n_e e^3 \ln \Lambda_0}{4\pi \epsilon_0^2 m_e c^2}$$

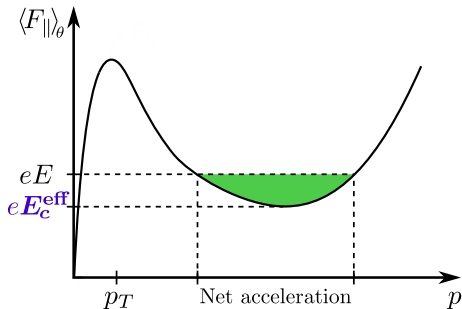


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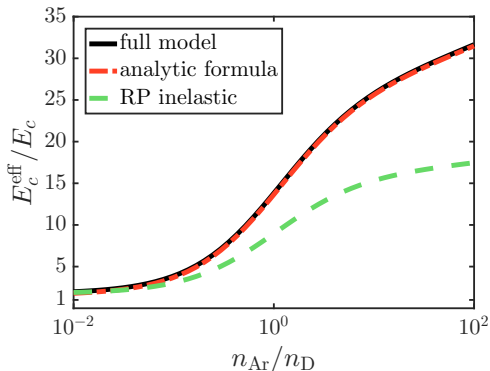
$$E_c = \frac{n_e e^3 \ln \Lambda_0}{4\pi \epsilon_0^2 m_e c^2}$$

- E_c enhanced by
 - Partially ionized atoms
 - Synchrotron radiation
 - Bremsstrahlung
 - Energy-dependent Coulomb logarithm $\ln \Lambda$



Enhanced critical electric field E_c^{eff}

- Large enhancement of E_c^{eff} due to partial screening
- Significant effect from elastic collisions
- RP model underestimates E_c^{eff}



$$\frac{E_c^{\text{eff}}}{E_c} \approx 1 + \frac{1}{\ln \Lambda_0} \left(7 - \ln \sqrt{T_{\text{eV}}} + 240 \frac{n_{\text{Ar,tot}}}{n_e} \right)$$

Derivation of E_c^{eff}

- Assume fast pitch-angle dynamics in Fokker–Planck equation:¹

$$\frac{\partial \bar{f}}{\partial t} = \frac{\partial}{\partial p} [(p\nu_S - eE\xi)\bar{f}] + \frac{\partial}{\partial \xi} \left[(1 - \xi^2) \underbrace{\left(\frac{eE}{pmc}\bar{f} + \frac{1}{2}\nu_D \frac{\partial \bar{f}}{\partial \xi} \right)}_{=0} \right]$$

where $\bar{f} = p^2 f$.

- Averaged force balance: $\langle eE_c^{\text{eff}} \rangle = \min_p p\nu_S$
- Up to triply ionized argon² $n_{\text{Ar}} \gtrsim 0.1 n_{\text{D}}$ (synchrotron neglected)

$$\frac{E_c^{\text{eff}}}{E_c} \approx 1 + \frac{1}{\ln \Lambda_0} \left(7 - \ln \sqrt{T_{\text{eV}}} + 240 \frac{n_{\text{Ar,tot}}}{n_e} \right)$$

¹Lehtinen et al, JGR (1999), Alevnikov and Breizman, PRL (2015)

²Hesslow et al, PRL (2017); Details in Hesslow et al, EPS (2017)

Simulate dissipation of runaway beam [1/2]

- Linear current decay predicted¹ : $-\frac{\partial j}{\partial t} \propto E \approx E_c^{\text{eff}}$
- Implemented in Fokker–Planck solver CODE with 0-D inductive electric field²

$$E = -\hat{L} \frac{\partial j}{\partial t}, \quad \hat{L} = \frac{AL}{2\pi R} \sim \frac{\mu_0 A}{2\pi}$$

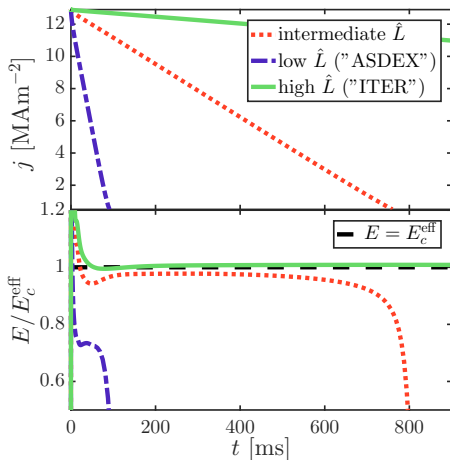
- Forward-beamed initial distribution obtained by simulation with large E-field, average runaway energy: 17.2 MeV

¹Breizman NF (2014)

²Wilkie et al in preparation; Stahl et al EPS P2.150

Simulate dissipation of runaway beam [2/2]

- Test $-\hat{L} \frac{\partial j}{\partial t} = E \stackrel{?}{\approx} E_c^{\text{eff}}$
- Good agreement at high inductance:
→ current decay rate is $\propto E_c^{\text{eff}} / \hat{L}$
- Enhanced $E_c^{\text{eff}} \Rightarrow$ faster dissipation
- Parameters: $T = 10$ eV, Ar^+ with $n_{\text{Ar}} = 4n_{\text{D}}$, $n_{\text{D}} = 10^{20} \text{ m}^{-3}$, initial average runaway energy 17.2 MeV



Summary: partial screening

Enhanced collision frequencies

- Analytical expressions for the deflection and slowing-down frequencies.
- Significant enhancement compared to complete screening, already at sub-relativistic electron energies.

Current decay time is reduced

- Low inductance case: current decay time is approximately half compared to the RP rule of thumb.
- High inductance case: current decay rate is $\propto E_c^{\text{eff}} / \hat{L}$

Critical electric field

$$\frac{E_c^{\text{eff}}}{E_c} \approx 1 + \frac{1}{\ln \Lambda_0} \left(7 - \ln \sqrt{T_{\text{eV}}} + 240 \frac{n_{\text{Ar,tot}}}{n_e} \right)$$

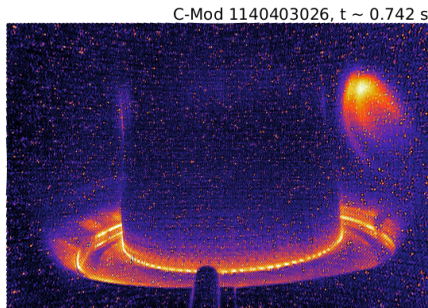
Highlights

- Recent papers
 - NORSE: A solver for the relativistic **non-linear** Fokker-Planck equation for electrons in a homogeneous plasma
[Stahl, Landreman, Embréus and Fülöp, CPC **212**, 269 (2017)]
 - Runaway-electron formation and electron **slide-away** in an ITER post-disruption scenario
[Stahl, Embréus, Landreman, Papp and Fülöp, JPCS **775** 012011 (2016)]
 - Effect of **partially ionized impurities** on fast electron dynamics
[Hesslow, Embréus, Stahl, DuBois, Papp, Newton and Fülöp, PRL **118**, 255001 (2017)]
- In preparation
 - SOFT: a synthetic **synchrotron diagnostic** for runaway electrons
[M Hoppe et al]
 - On the relativistic **large-angle** electron collision operator for runaway avalanches in plasmas [O Embréus et al]

4 SOFT

SOFT: Synthetic synchrotron diagnostics

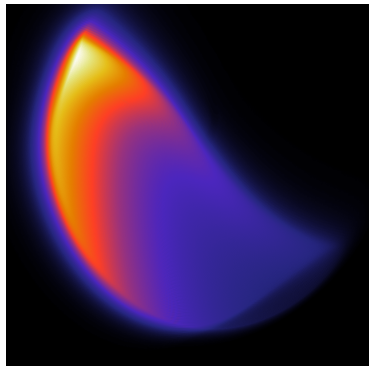
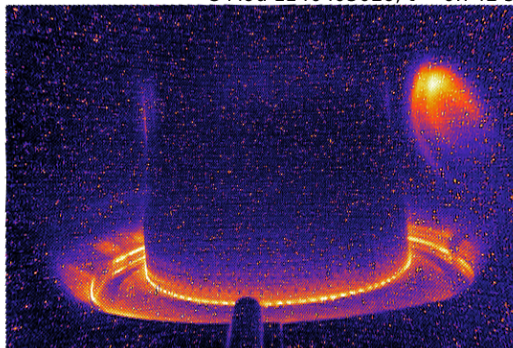
- SOFT – Synchrotron-detecting Orbit Following Toolkit
- Takes spectrum, camera location/size/viewing direction into account
- Uses experimentally obtained magnetic equilibria
- Solves the guiding-center equations of motion to distribute particles poloidally (accounts for geometric effects)
- Momentum-space distribution of runaways (e.g. obtained by CODE) given as input



Experimental image provided by A Tinguely and R Granetz

Strange synchrotron image? A case for SOFT!

C-Mod 1140403026, $t \sim 0.742$ s



M. Hoppe, et. al., *EPS 2017 conference*, (2017).

Spare slides

Heat-sink

- The total energy change can be written as

$$\frac{dW}{dt} = m_e c^2 \int_{\Omega} d^3 p (\gamma - 1) \left(-\frac{e\mathbf{E}}{m_e c} \cdot \frac{\partial f}{\partial \mathbf{p}} + \frac{\partial}{\partial \mathbf{p}} \cdot (\mathbf{F}_s f) + k_h \frac{\partial}{\partial \mathbf{p}} \cdot (\mathbf{S}_h f) \right)$$

from which k_h can be determined in each time step by demanding that $dW/dt = 0$.

- $\mathbf{S}_h(p)$ is an isotropic function of momentum (a natural choice is a Maxwellian).
- The momentum space need not necessarily encompass the entire population domain.
- In the figures Ω is the bulk of the distribution, which was defined as all particles with $v < 4v_{Th0}$ where v_{Th0} is the thermal speed at the initial temperature.